# Generative Models and EM

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MCB 112

# Outline

Generative models


EM

$$\hat{c}_k = \sum_{n=1}^{N} q_{nk} = \sum_{n=1}^{N} q_{nk}(\hat{\nu}_k)$$

$$\hat{\nu}_k = \frac{\hat{c}_k}{N}$$

# Outline

## Generative models


## EM

$$\hat{c}_k = \sum_{n=1}^{N} q_{nk} = \sum_{n=1}^{N} q_{nk}(\hat{\nu}_k)$$

$$\hat{\nu}_k = \frac{\hat{c}_k}{N}$$

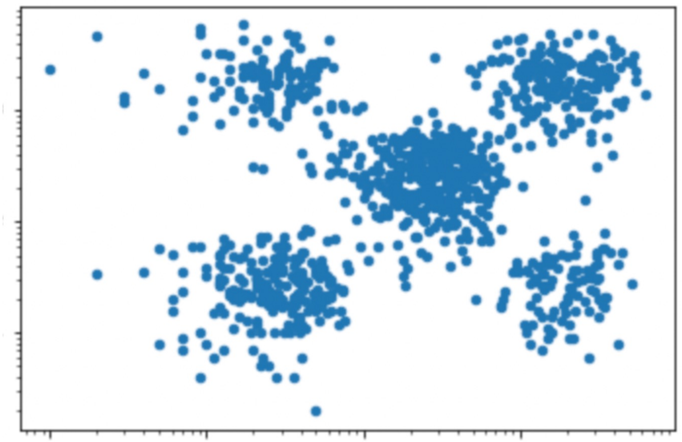
# Generative models

- variable and parameter types
- How each are generated

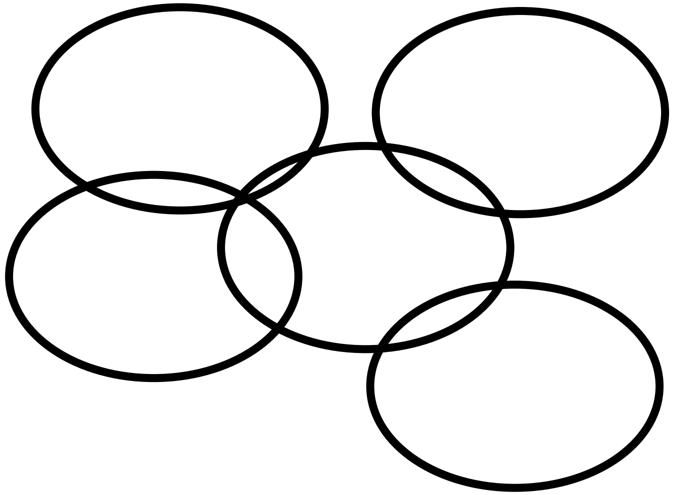
We will go through these with examples.


Graphical representation of a generative model.

# example: negative binomial fitting



data



generative model

# Generative models: parameter types

- Unknown parameters
- Known parameters
- Hidden variables
- Observed data

# parameter types: negative binomial fitting

- Unknown parameters: *centroids and mixture coefficients* $\mu_k, \pi_k$
- Known parameters: *dispersion* $\phi$
- Hidden variables: *group identity* $G_n$
- Observed data: *data points* $x_n \ (1 \leq n \leq N)$

# parameter types: RNA-seq experiment

- Unknown parameters: *nucleotide abundances* $\nu_1, \nu_2, \cdots, \nu_M$
- Known parameters: *transcript lengths* $L_1, L_2, \cdots, L_M$
- Hidden variables: *identity, orientation, start* $G_n, S_n, O_n$
- Observed data: *reads* $R_n \; (1 \leq n \leq N)$

# Generative models: specifications

- Which parameters are known and unknown?
- How do the parameters generate the hidden variables?
- How do the parameters and hidden variables generate the observations?

# Specifications: example (mixture NB)

- Which parameters are known and unknown?

$$\mu_k, \pi_k, \phi$$

- How do the parameters generate the hidden variables $G_n$?

$$P(G_n = j) = \pi_j$$

- How do the parameters and hidden variables generate the observations? $P(x_n | G_n = j) \sim \mathcal{NB}(\mu_j, \sigma^2)$

# Outline

Generative models




EM

$$\hat{c}_k = \sum_{n=1}^{N} q_{nk} = \sum_{n=1}^{N} q_{nk}(\hat{\nu}_k)$$

$$\hat{\nu}_k = \frac{\hat{c}_k}{N}$$

# EM

- Initialization
  - Make any guess for the unknown parameters.
- Expectation
  - use the generative model!
- Maximization
  - Find the unknown parameter values that maximize the likelihood.

# EM: negative binomial fitting

- Initialization
  - Make any guess for the unknown parameters $\mu_k, \pi_k$.

- Expectation
  - use the generative model!

$$P(G_n = j) = \pi_j$$

$$P(x_n | G_n = j) \sim \mathcal{NB}(\mu_j, \sigma^2)$$

  - Then find the posterior.

$$q_{nk} = P(G_n = k | x_n) = \frac{P(x_n | G_n = k) P(G_n = k)}{\sum_{j=1}^{K} P(x_n | G_n = j) P(G_n = j)}$$

# EM: negative binomial fitting

- Maximization
  - Find the values of the unknown parameters that maximize the likelihood.
  - In pset 5: rather than writing out the likelihood and then maximizing it, we took a point estimate by weighting with the posterior.

$$\hat{\mu}_k = \frac{\sum_{n=1}^{N} x_n \cdot q_{nk}}{\sum_{n=1}^{N} q_{nk}}$$

$$\hat{\pi}_k = \frac{1}{N} \sum_{n=1}^{N} q_{nk}$$

  - Now we have point estimates for the unknown parameters that we can plug back into the expectation step.

# EM: RNA-seq

- Initialization
  - Make a sensible guess for the unknown parameters $\nu_1, \nu_2, \cdots, \nu_M$.
  - Now you have a complete set of parameters $\boldsymbol{\theta}$.
- Expectation
  - use the generative model!

$$P(G_n = k|\boldsymbol{\theta})$$

What is the probability of generating a transcript with a certain identity?

$$P(R_n|G_n = k, \boldsymbol{\theta})$$

What is the probability of generating this read, given the transcript identity?

  - Then find the posterior.

$$q_{nk} = P(G_n = k|R_n, \boldsymbol{\theta}) = \frac{P(R_n|G_n = k, \boldsymbol{\theta})P(G_n = k|\boldsymbol{\theta})}{\sum_{j=1}^{K} P(R_n|G_n = j, \boldsymbol{\theta})P(G_n = j|\boldsymbol{\theta})}$$

# EM: RNA-seq

- Maximization
  - Find the values of the unknown parameters that maximize the likelihood.
  - Again, we take a point estimate by ”weighting” with the posterior.

$$\hat{c}_k = \sum_{n=1}^{N} q_{nk} = \sum_{n=1}^{N} q_{nk}(\hat{\nu}_k)$$

$$\hat{\nu}_k = \frac{\hat{c}_k}{N}$$

  - Now we have point estimates for the unknown parameters that we can plug back into the expectation step.

# Resources for this week

- In these slides, there’s a sketch of how to do pset part 3.
- You can refer to past resources to see how EM was implemented for negative binomial fitting.
- Section notes contains a sketch of how to write an array for calculating the expectation.